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The Valuation of Public Goods: Why Do We Work?

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Nicholas Flores
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

Philip E. Graves
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

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Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

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Nicholas E. Flores
Department of Economics 256 CUB
University of Colorado
Boulder, CO 80309-0256
e-mail nicholas.flores@colorado.edu

Philip E. Graves
Department of Economics 256 CUB
University of Colorado
Boulder, CO 80309-0256
e-mail: philip.graves@colorado.edu

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Philip E. Graves

Abstract

Conventional analysis of public goods provision aggregates individual willingness to pay while treating income as exogenous. This process ignores the fact that we generate income to allow us to purchase utility-generating goods. From the individual perspective, there exists an optimal level of income generated that would be associated with an optimal level of public goods. To the extent that there is an output market failure in the provision of public goods, there will also exist an accompanying input failure in labor supplied/leisure demanded. We explore the implications of this input market failure by explicitly considering leisure demand in a model of public goods provision. We consider two areas of public goods analysis, benefit analysis and optimal provision. Based on our analysis, we conclude that by misinterpreting the basic nature of income, conventional analysis generally leads to an incorrect provision of public goods.

[1]Musgrave (1969) raises the problem that one cannot separate issues of optimal public goods provision and optimal distribution of wealth. Bergstrom and Cornes (1983) provide a general class of preferences that avoid Musgrave's problem. Flores (forthcoming) explores the relationship between income distribution and willingness to pay in the context of non-paternalistic altruism.

[2]The double divided is the potential welfare gain that may occur when revenues from pollution taxes are used to reduce distortionary labor taxes. For an overview of this literature see Parry and Oates (2000).

[3] Another area where researchers have considered labor supply decisions is with regard to recreational demand literature. The emphasis in this literature has been on inferring a more precise value of time that can then be used in

public goods.[5] In a very real sense, there is a heretofore unrecognized input market failure that corresponds to the output market failure: inability to *individually* buy the public good will result

[5]Samuelson assumes this interconnectedness away: "Provided economic quantities can be divided into two groups, (1) *outputs* or goods which everyone always wants to maximize and (2) inputs or factors which everyone always wants to minimize, we are free to change the algebraic signs of the latter category and from then on to work only with 'goods,' knowing that the case of factor inputs is covered as well."

[6]Here we are not claiming that *all* individuals have these same Cobb-Douglas preferences or even Cobb-Douglas preferences at all. Rather we are simply using these preferences to explain the relationship between public goods and income generated..

[7]The curves are similar in that they are given by the relevant ! utility parameter divided by the price multiplied

[8] In this case, $(H - L^{0})/(H - L^{*}) = [H - (H!_{L})/(!_{X} + !_{L})]/[H - (H!_{L})/(!_{X} + !_{L} + !_{q})] =$ $[!_{X}/(!_{X} +$.

Sten's true willingness to pay, *if he could purchase public goods in the same way he can private goods*, would be much larger than his apparent willingness to pay at the low income he generates in a world in which he cannot so purchase public goods. After all, Sten is poor because he optimally chose not to work. In a very real sense, the *same* market failure that causes non-optimal provision of public goods in the market setting also creates a market failure in input markets: the failure to generate income when that income cannot buy what we would like.

3. Benefit Analysis and Input Market Failure

Standard welfare analysis for valuing a change in a quantity rationed public good typically treats income as being fixed. This implicitly assumes that leisure demanded/labor supplied, is constrained to the status quo level. It generally is true that if we constrain a person to the same level of leisure, then the gain in utility for an increase in the public good will be less than if we allow for adjustment in leisure. In turn, the compensating variation/willingness to pay for the increase under constrained leisure choices will be less than willingness to pay under a regime of free choice of leisure. Similarly for the same reduction, equivalent variation/willingness to accept compensation will also be less than if we had allowed for leisure adjustment.

Our general model is very similar to standard models of public goods provision. A representative consumer's preferences are defined over a vector of market goods, X, a public good, q, but now we add leisure, L. Though we use a representative agent in developing our analytical points, it is not critical that everyone have identical preferences. We focus on a representative consumer because our observations need only be discussed in the context of a single individual; the main points carry directly over to multiple consumers. While our model is similar in spirit to the typical public goods analysis, the addition of leisure differs from the

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$L^h = L^h(p, w, ", q, U)$.[11] Using the demands, we can easily represent the amount of wealth adjustment that would leave our consumer indifferent between obtaining an increase in the public good from an initial level q^0 to a new, higher level q^1. As in standard welfare analysis that treats income as exogenous, the wealth adjustment that makes the consumer indifferent is referred to as compensating variation and is equal to the difference in the minimized expenditures.

$$\begin{aligned} CV &= p \cdot [X^h(p,w,q^0,U^0) - X^h(p,w,q^1,U^0)] \\ &\quad + w \cdot [L^h(p,w,q^0,U^0) - L^h(p,w,q^1,U^0)] + "[q^0 - q^1] \end{aligned} \qquad \textbf{(3)}$$

In order to contrast (3) with a standard welfare analysis that treats leisure demand as fixed, we constrain the choice of our consumer while providing the same increase in the public good. The minimization problem will be identical to the problem in (2) with the exception that leisure is constrained to the status quo level of leisure which we will refer to as L^0. Now the only choice variables in the expenditure minimization problem are the levels of the market goods, *X*. Given the constraint we alter our notation to reflect this constraint, $X_S^h = X_S^h(p, w, L^0, ", q, U^0)$, letting the *S* subscript refer to the "standard" notion of compensated demand where leisure is fixed. With our new notation, we can express the standard notion of compensating variation found in the welfare economics literature.

$$\begin{aligned} CV_S &= p \cdot [X_S^h(p,w,L^0,",q^0,U^0) - X_S^h(p,w,L^0,",q^1,U^0)] \\ &\quad + w \cdot [L^0 - L^0] + "[q^0 - q^1] \end{aligned} \qquad \textbf{(4)}$$

As our intuition suggests, the relationship between the standard compensating variation (4) and compensating variation with flexible leisure demand (3) is easy to establish.

[11] The *h* superscript refers to the solution to the dual problem, the compensated or Hicksian demands.

Proposition 1: For an increase in the public good from q^0 to a new higher level q^1, it will generally be true that $CV_S \leq CV$. Furthermore, in most cases $CV_S < CV$.

Proof: *Part 1.* The first statement in proposition 1 is very easy to establish. First note that by definition $p \cdot X^h(p,w,",q^0,U^0) + w \cdot L^h(p,w,",q^0,U^0) + "q^0 = p \cdot X_S^h(p,w,L^0,",q^0,U^0) + w \cdot L^0 + "q^0$;

optimal provision which must account for utility change. We show that when income is treated as exogenous, under provision will generally result even when using a Samuelson price scheme.

4.1. Efficient Pricing with Leisure Demand

The basic model for this section is the same as in section 3. Restating, the consumer's problem is to maximize utility in the choice of market goods, *X*, and leisure, *L*. The public good, *q*, is quantity rationed. The marginal tax rate charged to the consumer for *q* is $''$. The maximization problem is as follows.

$$\max_{X,\ L} \ U(X,\ q,\ L) \quad s.t. \quad pX + wL + ''q = wH,\ q = q^{0} \qquad \textbf{(5)}$$

In order to discuss efficient pricing of the public good, it is useful to write out the Lagrangean for this problem and the ensuing first order conditions.

$$\mathcal{L} = U(X,\ q,\ L) + \#(wH - pX - wL - ''q) + \$(q^{0} - q) \qquad \textbf{(6)}$$

An efficient price consistent with Samuelson's characterization requires that $''$ is such that the marginal value of relaxing the rationing constraint, $\$$, exactly equals zero. If we were to determine the efficient price schedule, $''(q)$, that would induce our individual to want to demand that specified level of *q* for all *q*, we would recover the inverse demand curve. Optimal provision would require the sum of these inverse demand curves to exactly equal the marginal cost of provision which is a form of modified Samuelson rule that explicitly recognizes that labor supply is an integral part of the problem.

4.2. Marginal Valuation with Leisure Demand

A problem with an inverse demand approach is that when *q* is not matched with the efficient price, we are essentially observing demand for *X* and *L* along demands that are conditioned on the level of *q*. Furthermore, the rationed level of *q* is not along the demand curve since the rationing constraint is binding. The companion to the inverse demand is the marginal valuation function. For a given rationing level, the marginal value of more *q* is simply the marginal utility of *q* scaled by the marginal utility of income.[12]

$$p^{q}(q \mid p, w, ") \;=\; \frac{\partial U(X,\ q,\ L)}{\partial q}\,\frac{1}{\#} \qquad \textbf{(7)}$$

The values for *X*, *L*, and $\#$ are the solutions to the rationed problem for the specified level of *q*. Of course increasing *q* comes at a marginal cost of $"$. Thus the net marginal value which equals $\$/\#$ is simply the marginal value minus $"$.

$$NMV \;=\; p^{q}(q \mid p, ", w) \;-\; " \qquad \textbf{(8)}$$

At an efficient price $"$, the net marginal value exactly equals zero. In this case, the marginal valuation curve exactly equals the inverse demand. However when the net marginal value is greater than zero, the marginal valuation curve diverges from the inverse demand at that particular value of *q*.

4.3. Marginal Valuation with Exogenous Income

As noted above, the standard analysis treats income as exogenous, modeling preferences only over market goods and the public good. The problem is presented as maximizing utility in

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[12] The marginal valuation curve will also depend on the prices of market goods and the tax rate for *q*. We are suppressing these arguments to economize on notation.

the choices of *X* by allocating money among the *X* goods subject to a rationed level of *q*.

$$\max_{X} \; U(X,\ q) \quad s.t. \quad pX \ + \ "q \ = \ y, \ q \ = \ q^{0} \tag{9}$$

Translating back to our model where preferences also include leisure, the problem in (9) is obtained by rationing leisure which results in the same initial level of income, $y^{0} \ = \ wH \ - \ wL^{0}$, where L^{0} is leisure demanded at rationed level q^{0}. As above, treating income as exogenous when it is not is equivalent to assuming the consumer solves the following problem.[13]

$$\max_{X} \; U(X,\ q,\ L) \quad s.t. \quad pX \ + \ wL \ + \ "q \ = \ wH, \ q \ = \ q^{0}, \ L \ = \ L^{0} \tag{10}$$

$$p^{s}(q \mid p, w, L, ") \ = \ \frac{\partial U(X,\ q,\ L)}{\partial q} \frac{1}{\#^{s}} \tag{11}$$

[13]Hanemann and Morey (1992) consider the consequences of estimating an incomplete demand system on calculating welfare measures. While our work is related in that some choice variables are being ignored in the analysis, the implication is more fundamental for our work because income is not just ignored, but rather the wrong income is used in the analysis.

$$NMV^{s} = p^{s}(q|p,w,L,") - " \qquad \textbf{(12)}$$

At q^0, the marginal valuation curves from the leisure choice approach and the income exogenous approaches are equal as are the net marginal valuation curves. However when q is increased, the two will diverge due to the constraint implicitly imposed on leisure when treating income as exogenous. The general result we seek to prove is that for $q > q^0$, $p^{s}(q|p,w,L^0,") < p^{q}(q|p,w,")$. To this end we must introduce compensated valuation curves.

4.4. Compensated Marginal Valuation

Recall the dual utility-constant formulation from above.

$$\min_{X,\ L}\ pX + wL + "q \quad s.t.\ U(X,\ q,\ L) = U^0,\ q = q^0 \qquad \textbf{(13)}$$

The Lagrangean for this problem is given as follows.

$$\mathcal{L} = pX + wL + "q + \#_c(U^0 - U(X,\ q,\ L)) + \$_c(q - q^0) \qquad \textbf{(14)}$$

The respective marginal value and net marginal value functions for this problem are given below.

$$p^{v}(q|p,w,",U) = \frac{\partial U(X,\ q,\ L)}{\partial q}\#_c \qquad \textbf{(15)}$$

$$NMV = p^{q}(q|p,w,",U) - " \qquad \textbf{(16)}$$

At q^0, all the marginal valuation functions are equal at the initial reference utility level.

$$p^{q}(q^{0}|\, p,w,") \;=\; p^{s}(q^{0}|\, p,w,L^{0},") \;=\; p^{v}(q^{0}|\, p,w,",U^{0}) \qquad \textbf{(17)}$$

At an increased level of *q*, say q^{1}, we know that moving along both uncompensated marginal valuation curves, utility is increasing when the good is under-provided. Given the constraint on leisure when treating income as exogenous, we know that the utility gain in the leisure restricted case is bounded above by the utility gain obtained when leisure is not restricted. Let U^{1} represent the utility level obtained when leisure is not restricted and $U^{/}$ the utility gain when leisure is restricted. Further assume that *q* is a normal good in the sense that if we provided additional non-labor income, demand for *q* would increase if the consumer did not face the ration constraint on *q*. Under strict convexity of preferences, $U^{0} < U^{/} < U^{1}$. The compensated marginal valuation function can be used to match the marginal valuation curves that are uncompensated. For the case of moving along the curve where leisure is not restricted, we know the following.

$$p^{q}(q^{1}|\, p,w,") \;=\; p^{v}(q^{1}|\, p,w,",U^{1}) \qquad \textbf{(18)}$$

Given the assumption of normal goods, $p^{v}(q|\, p,w,",U^{0}) < p^{v}(q|\, p,w,",U^{1})$ for all *q* which implies that $p^{q}(q^{0}|\, p,w,") < p^{q}(q^{1}|\, p,w,")$. Now consider $p^{s}(q^{1}|\, p,w,L^{0},")$. In terms of the compensated marginal value, there does exist a compensated problem that would exactly support the choice of market goods and leisure that is realized under the fixed leisure problem. In order to keep the choice of leisure fixed at L^{0}, a different wage would be required to keep leisure at the initial level. We refer to that wage as $w^{/}$ and note that $p^{s}(q^{1}|\, p,w,") = p^{v}(q^{1}|\, p,w^{/},",U^{/})$. From the assumption of normal goods, $p^{v}(q^{1}|\, p,w,",U^{/}) < p^{v}(q|\, p,w,",U^{1})$. The question is one of what is the effect of the wage change. We need to consider two cases.

Case 1: Compensated Substitution Between q and L

Compensated substitution can be expressed in several equivalent ways according to the rationing classifications of Madden (1991). One way of defining substitution is as the case when the compensated demand for

$$\begin{aligned} p^{s}(q^{1}|,p,w,L^{0},'') = \\ p^{v}(q^{1}|p,w^{/},'',U^{/}) < p^{v}(q^{1}|p,w,'',U^{/}) < p^{v}(q|p,w,'',U^{1}) \\ = p^{q}(q^{1}|p,w,'') \end{aligned} \qquad \textbf{(19)}$$

$$p^{s}(q^{1}|,p,w,L^{0},") =$$

$$p^{v}(q^{1}|\,p,w^{/},",U^{/}) < p^{v}(q^{1}|\,p,w,",U^{/}) < p^{v}(q|\,p,w,",U^{1}) \qquad \textbf{(20)}$$

$$= p^{q}(q^{1}|\,p,w,")$$

Proposition 3: Suppose that preferences are strictly convex, *q* is a normal good, and that *q* and *L* are compensated complements. Then for all $q > q^{0}$, $p^{s}(q|\,p,w,L^{0},") < p^{q}(q|\,p,w,")$.

From propositions 2 and 3, we find that the marginal valuation curve treating income as exogenous lies below the true marginal valuation curve. Thus using the income exogenous valuation curve to determine optimal provision will generally result in under provision relative to the true optimum.

5. Discussion of Implications and Extensions

If we cannot acquire the goods we desire, we lose the incentive to work. In the case of public goods, the degree of this effect is determined by the relative strength of individual preferences for private versus public goods. Collectively, we will under-supply work effort, generate too little income, and under-supply the public good.

These work-leisure choices extend to the realm of human capital accumulation. Individuals, caring in varying degrees about the environment, make human capital decisions over their life cycles, as well as making the leisure/labor decisions in each period emphasized here. Each will choose to accumulate different levels of human capital, depending on expected levels of public goods provision. For some, private demands for the public good will be only modestly greater than the social provision level, hence only minor adjustments in human capital decisions and leisure/goods consumption might occur. This may well be a typical situation among those not characterizing themselves as "environmentalists."

Others, like Sten, will have demands for environmental quality far in excess of the level

rough partial equilibrium guess at the “proper” amount of environmental goods to be produced is

[18]The text numbers would be reduced some in general equilibrium as depicted in Figure 2.

6. References

Bergstrom, T. C. and R. C. Cornes (1983). "Independence of Allocative Efficiency from Distribution in the Theory of Public Goods." Econometrica **51**(6): 1753-1765.

Bockstael, N. E., I. E. Strand and W. M. Hanemann (1987). "Time and the Recreational Demand Model." American Journal of Agricultural Economics **69**(2): 293-302.

Clarke, E. H. (1971). "Multipart Pricing of Public Goods." Public Choice **11**: 17-37.